

1.3 | 연습문제 정답

01. $\frac{29}{12}$

02. $\cosh(-x) = \frac{e^{-x} + e^x}{2} = \frac{e^x + e^{-x}}{2} = \cosh x$

03. $\cosh x - \sinh x = \frac{e^x + e^{-x}}{2} - \frac{e^x - e^{-x}}{2} = e^{-x}$

04. $1 - \tanh^2 x = 1 - \frac{\sinh^2 x}{\cosh^2 x} = \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x} = \frac{1}{\cosh^2 x} = \operatorname{sech}^2 x$

05. $\coth^2 x - 1 = \frac{\cosh^2 x}{\sinh^2 x} - 1 = \frac{\cosh^2 x - \sinh^2 x}{\sinh^2 x} = \frac{1}{\sinh^2 x} = \operatorname{csch}^2 x$

06. $\sinh x \cosh y + \cosh x \sinh y = \left(\frac{e^x - e^{-x}}{2} \right) \left(\frac{e^y + e^{-y}}{2} \right) + \left(\frac{e^x + e^{-x}}{2} \right) \left(\frac{e^y - e^{-y}}{2} \right)$
 $= \frac{e^{(x+y)} - e^{-(x+y)}}{2} = \sinh(x+y)$

07. $\cosh x \cosh y + \sinh x \sinh y = \left(\frac{e^x + e^{-x}}{2} \right) \left(\frac{e^y + e^{-y}}{2} \right) + \left(\frac{e^x - e^{-x}}{2} \right) \left(\frac{e^y - e^{-y}}{2} \right)$
 $= \frac{e^{(x+y)} + e^{-(x+y)}}{2} = \cosh(x+y)$ 이다.

08. $\tanh(x+y) = \frac{\sinh(x+y)}{\cosh(x+y)} = \frac{\sinh x \cosh y + \cosh x \sinh y}{\cosh x \cosh y + \sinh x \sinh y}$ 이므로
 $\cosh x \cosh y$ 로 분모, 분자를 나누면 $\tanh(x+y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$ 이다.

09. e^{2x}

10. $\cosh x + \sinh x = e^x$ 이므로 $(\cosh x + \sinh x)^n = e^{nx} = \cosh nx + \sinh nx$

11.
$$\begin{aligned} \sinh(x+y)\sinh(x-y) &= \left(\frac{e^{(x+y)} - e^{-(x+y)}}{2}\right)\left(\frac{e^{(x-y)} - e^{-(x-y)}}{2}\right) \\ &= \left(\frac{e^{2x} + e^{-2x}}{4}\right) - \left(\frac{e^{2y} + e^{-2y}}{4}\right) = \left(\frac{e^{2x} + e^{-2x} - 2}{4}\right) - \left(\frac{e^{2y} + e^{-2y} - 2}{4}\right) \\ &= \sinh^2 x - \sinh^2 y \end{aligned}$$

12.
$$\begin{aligned} 2\left(\cosh \frac{x}{2}\right)^2 - 1 &= 2\left(\frac{e^{\frac{x}{2}} + e^{-\frac{x}{2}}}{2}\right)^2 - 1 = 2\left(\frac{e^x + 2 + e^{-x}}{4}\right) - 1 = \cosh x \text{ 이므로} \\ \cosh \frac{x}{2} &= \sqrt{\frac{\cosh x + 1}{2}} \end{aligned}$$

13. $\frac{13}{12}$

14. $\pm \frac{4}{3}$

15. $-\frac{1}{2}$